



Zeno of Elea's Paradox, Dichotomy, as the Origin of Atomic Theory. A New Solution to Dichotomy based on Planck's Action Constant

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Abstract A very complete solution to one of Zeno of Elea's paradox, Dichotomy, has been found. Our solution is based on two complementary physical magnitudes, energy and time. Moreover, the hypothesis is advanced that a part of our reasoning could be a lost paradox of Zeno of Elea. Solutions of other authors to Dichotomy are also presented. By putting a limit to infinite divisibility of space and time, when applied to matter, Leucippus and Democritus were led to atomic theory in its incipient form.

Keywords Zeno of Elea, paradox, aporia, Planck's constant, atomic theory

1. Introduction

There is a time to imagine paradoxes and a time to solve them. In-between scholars regard them as wild beasts, dangerous for thinking and logical systems. After they are solved (tamed) people consider them with admiration, use or even abuse them.

Zeno of Elea (ca 490-430 BC), one of the most brilliant philosopher of antiquity, was a representative of Eleatic School. He and Socrates are considered the inventors of a philosophical current, dialectic [1,2]. In order to defend the philosophy of his master Parmenides (born c. 515 BC), Zeno of Elea imagined a number of 40 paradoxes (aporiae) [3], included in a book presented on the occasion of his visit to Athens. Zeno's book got lost, and only a few paradoxes (less than ten) survived to our days, although not by his writings. Four of them are extremely important: Dichotomy, Achilles and the Tortoise, the Arrow and the Stadium [4]. Zeno of Elea's paradoxes came down to us in two ways: on the one hand, through the writings of Melissos of Samos and Simplicius [5], who, having made a fair copy of Zeno's writings transmitted them as such, and on the other hand, through the writings of Aristotle [5] and Plato (427-347 BC) [6] who "translated" them in their own jargon thus inevitably introducing alterations. It has to be mentioned that the aporiae transmitted by Aristotle and Plato are the most important by their content.

Parmenides is considered the founder of Eleatic School although its beginner is Xenophanes (c. 570-c. 478 BC). Xenophanes' philosophy is included in a few books. Concerning the essence of his ideas, Xenophanes maintained absolute existence to be the One, and this One is involved in all things. At the same time, it is unchangeable, without beginning, middle or end. A fundamental principle of Xenophanes was that thought and reason are everything and eternal, and by this assertion he denied the truth of the conceptions of origination and of passing away, of change, movement, etc., considering that they merely belong to sensuous perception, they are only semblance. The finding



of pure thought by Xenophanes constituted a remarkable advance, and thought thus becomes for the first time independent for itself in the Eleatic School. A double consciousness can be found in the philosophy of Xenophanes: a consciousness of Being and a consciousness of opinion [1].

Parmenides, a former disciple of Xenophanes, completed the conceptions of his master. The opposition between being and non-being appears still more clearly to Parmenides. According to Parmenides, the things are as they are: they cannot be neither gathered nor scattered; many things and phenomena which seems quite evident to us, such as motion, change, and diversity, are simply illusions and the reality is in fact an absolute, unchanging One. In this way, Parmenides placed himself in a zone of distrust, no wonder that his disciples had to defend his assertions. However, some philosophers credited Parmenides with a great and unique contribution to the nature knowledge, i.e. the circular dimension of reality ("It is all one to me where I begin; for I shall come back again there") (it's an idea which can be found in Ecclesiastes), contrary sides (day-night; it is, it is not; the same, not the same; etc.). Parmenides introduced into Western philosophy the great conception of a reality behind the passing illusions of sense, i.e. a reality one, indivisible, and unchanging, on the basis of a logical argument as to the impossibility of not-being [1].

In this paper a new physical solution to one of Zeno of Elea's paradox, Dichotomy, is presented, in comparison with other mathematical and physical solutions.

2. Dichotomy Paradox of Zeno of Elea and its Solutions

Dichotomy paradox of Zeno of Elea denied the existence of movement in universe. Dichotomy paradox stated: before a body in motion, situated initially in the point A, can reach a given point B, it must first traverse the half of the AB distance; before it can traverse the half of the AB distance, it must first traverse the quarter; and so on *ad infinitum*. Thus the body will neither traverse the distance AB nor move from its place [3,6-10].

By putting a limit to divisibility [11], and calling this limit atom, Leucippus (c. 480 BC-c. 420 BC) and Democritus (c. 460-370 BC), the chemical philosophers [12], arrived to the well known atomic theory in its incipient form.

A renown physicist asked himself rhetorically what phrase – of all phrases ever written or uttered, is the richest in information. The answer was given by the same physicist and it sounded in this way: world is formed of atoms [13], in other words, a very condensed form of atomic theory. This "dialog" of Feynman is an excellent illustration of the importance of atomic theory. This theory is undoubtedly a collective achievement and one of the most brilliant accomplishment of mankind.

Few facts are known about Leucippus. So few that Epicurus denied his existence. However, his citation by Aristotle and by others determined the recognition of his existence. He was much influenced by Parmenides and Zeno [2]. Numerous facts are known about Democritus. He travelled to Athens, Egypt and Mesopotamia, and probably to India. In his time, Democritus had acquired also a reputation as a geometer. He boasted that not even the "rope-stretchers" in Egypt excelled him in mathematics. He pretended to be a Pythagorean but his actions indicate him as a product of Eleatic School. Democritus imagined a paradox of excellent resemblance with Zeno's paradoxes. This paradox is much less commented than Zeno's paradoxes, however it discloses a genius of the same quality. According to Democritus's paradox, a cone (or a pyramid) is cut in very thin slices parallel to the basis. Now let's compare the slices concerning their area. If they are equal, the cone is no more a cone but a cylinder (or the pyramid is no more a pyramid but a prism). If the slices are different, the reconstitution of the initial body fails, and a step cone (or a step pyramid) devoid of smoothness is obtained [12].

In fact, the philosophy of Eleatic School denied movement, change and diversity. Concerning Dichotomy, it should be established what kind of motion is analyzed. Some authors [4] indicate material points, other means photons by indicating light [14], other means runner [15] and other authors speak about motion without indicating the moving object [16]; probably Russell's term is the most suitable, and consequently it's our option.

Two types of solutions have been found till now to the Dichotomy paradox of Zeno of Elea: (A) mathematical solutions and (B) physical solutions. Some of them are simply comments, hence difficult to be considered solutions.



Our solution is of the second type. Some physical solutions use elements of relativistic theory, other quantum mechanics.

Aristotle (384-322 BC) opposed Zeno's Dichotomy paradox showing that the space can be divided *ad infinitum* but it is not divided like that [6]. Diogenes of Sinope (c. 404-323 BC) opposed the arguments about the immanent contradiction of motion by simply walking, an argument considered as superficial by Hegel who stated that we should not be content with the sensorial certitude but also should understand the matter [1]. Referring to the method used by Zeno –*reductio ad absurdum*– Hegel stated as follows: we should prove a statement to be false or true by placing ourselves inside it and not only by proving that the adverse statement is true or false.

2.1. Mathematical Solutions

The brightest mathematical solution has been elaborated by Cantor. Important contributions to the subject brought Cauchy, Dedekind and Weierstrass [3,4,10,17]. The distance between the two positions A and B is considered to be equal to the unit. Adding them up we get:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$$

Thus for a great number of divisions the summing up tends to the unit which is the very distance AB [10,18]. In fact all mathematical solutions gravitate around the contributions of Cantor. McLaughlin [19] use infinitesimals in a theory called by them internal set theory (IST). They also invent a new paradox, the falling apple. McLaughlin considers Dichotomy arguable and refute this paradox by the epistemological principle that we are not responsible for explaining situations we cannot observe. Moreover, they mention, Zeno's infinite series of checkpoints contains nonstandard numbers, which have no numerical meaning. Groarke [16] clearly rallies to the solution given by Cantor. A merit of Groarke's paper, in our opinion, is to notice a vague relationship between Dichotomy paradox and energy.

2.2. Physical Solutions

Lynds [17] accepts the solution developed by Cauchy, Weierstrass, Dedekind and Cantor in a strictly mathematical sense. At the same time, he uses relativistic mass and relativistic momentum as well as notions of quantum mechanics in order to solve Dichotomy paradox of Zeno of Elea. A solution came from the application of Heisenberg's uncertainty principle to the time and space divisions suggested by Zeno's Dichotomy. However, the latter notions involve microcosmic dimensions, and not microscopic as Lynds prefers [20]. An excellent relativistic solution to Zeno's Dichotomy aporia was found by Brown [14]. This solution has been presented in two variants: (A) a system of an infinite number of opposed mirrors at distances decreasing with the ratio 1/2; a flux of photons falling on the first mirror will traverse the whole system; (B) a system of squares, with diagonals decreasing with the ratio 1/2 and distributed on a spiral have to be diagonally traversed by light. In both cases all mirrors, although infinite in number, will be traversed in a finish time [11,14].

2.3. Our Physical Solution

There is a perfunctory aspect of Zeno's Dichotomy aporia: motion can be neither described nor accomplished exclusively in terms of space and time. We have asked ourselves, by paraphrasing Cajori [9], how is it possible to ignore energy and time in questions involving motion? In this paper we suggest a way of solving by using the concepts of physics. Zeno's paradox could be solved by using the quantification of matter and energy. One may suppose that one of Zeno's paradoxes sounded in this way: it's impossible to empty a recipient containing a liquid because in order to completely empty it one must first empty half of it, in order to empty half of it one must empty a quarter and so on and so forth *ad infinitum*. But today we know that this division *ad infinitum* does not really reach the infinitum[11]; it only reaches the level of molecules (the smallest particles in which the physical-chemical



properties of the liquid are to be found entirely) [21]. Consequently, as far as we can remove all the molecules, the vessel can be emptied out of the liquid.

On the other hand motion of a body (material point), in Zeno's terms, from position A to position B assumes not only time and space but time and energy, the two complementary physical magnitudes of physics [13,22]. Hence, the motion of a body from position A to position B presumes the energy E and is achieved in time t. Their multiplication, $E \times t$, constitutes the physical magnitude called action, A:

$$E \times t = A$$

And it is known from physics that the action is quantified [13,22]:

$$A = n \times h$$

in which h is the constant of action, Planck's constant, and n is a natural number. Therefore the motion of a mobile from position A to position B presumes the achievement of a natural number of action quanta. And the problem is similar to the emptying out of a vessel containing a liquid.

Concerning number n it can be a very big number and a natural one. There is a resemblance to the assertion that an amount of an element or chemical compound, some moles in size, that is a macrocosmical quantity, contains a very large number of atoms or molecules (microcosmic amounts), respectively, Avogadro's number, 6.023×10^{23} , adequately multiplied [21].

3. Conclusions

1. A new solution to one of Zeno's paradox, Dichotomy, based on Planck's action constant, has been presented.
2. Leucippus and Democritus applied to matter Zeno's divisibility of time and space, and put a limit, called atom, to this divisibility. In this way, atomic theory in its incipient form appeared.
3. Both Leucippus and Democritus belong to Eleatic School, according to their writings and ideas.

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